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# A Reaction-Diffusion Framework for Modeling the Dynamics of Insurgency in Nigeria: A Deterministic Compartmental Approach with Spatial and Relapse Effects

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**Abstract**: Insurgency remains a persistent challenge in developing nation Nigeria, where socio-political and economic factors fuel cycles of violence and rebellion. In this study, we develop a deterministic mathematical model to describe the spatiotemporal dynamics of insurgency. The population is partitioned into four interacting compartments; vulnerable individuals, active insurgents, security personnel, and reformed individuals. The model incorporates some important features such as ideological luring, recruitment through peer and cross-class influence, reform mechanisms, relapse into insurgency, and tactical neutralization. Spatial diffusion terms are included to simulate mobility and regional spread via a reaction-diffusion system of partial differential equations. We employ the Finite Difference Method hybridized with Method of Lines (MOL) time-stepping to discretize and simulate the nonlinear system using realistic variable and parameter values associated to terrorism in Nigeria. The model accounts for boundary conditions representing no external migration, and initial conditions are defined spatially. The model provides better understanding into the roles of vulnerability management, deradicalization, and security interventions in mitigating insurgency over time and space in Nigeria.
**Keywords:** Vulnerability dynamics, Deradicalization, Spatial spread, Relapse mechanism, Compartmental model, Finite difference method

## 1. Introduction

The rise and persistence of insurgency in Nigeria have posed severe threats to national security, economic development, and societal cohesion. Rooted in complex socio-political, economic, and ideological factors. Insurgency manifests through violent activities, recruitment of vulnerable individuals, and the use of territories or hideouts to advance rebel agendas, [11]. The persistence of such insurgent groups, particularly in the northeastern and north-central regions demands rigorous analytical approaches for understanding, predicting, and controlling their spread.
In recent years, mathematical modeling has emerged as a powerful tool in analyzing terrorism and insurgency. Also, [5-7] proposed a deterministic framework capturing the behavioral dynamics of terrorism and probed into how control measures like technological intelligence can influence radicalization trajectories. Similarly, [3] modeled the Niger Delta insurgency by integrating political grievances and socio-economic dissatisfaction into the simulation of rebel behavior. More recent contributions have focused on modeling banditry and insurgency-control interactions. For instance, [14] developed a banditry mathematical model incorporating optimal intervention strategies, while [1] incorporated counseling and rehabilitation in an insurgent-security interaction modeling framework.
Other models have expanded the modeling landscape to include arms proliferation [9], the psychological and media influence on terrorism [12], and control strategies using drones, [6]. In addition, [4] introduced insurgency effects into tuberculosis dynamics, emphasizing the interconnectedness of health and security crises. Several models have also dealt with relapse and recycling among ex-insurgents. [8] incorporated backward bifurcation to capture terrorist recycling, while [13] applied fractional calculus to assess long-term counterterrorism outcomes. In addition, [2, 10] proposed deterministic models for anti-banditry and terrorism control respectively by integrating community-level vulnerability and enforcement dynamics. [16] linked farmer-herder conflicts to biological vectors (tsetse flies) to show the cross-disciplinary complexity of insurgency-prone environments. Also, [15] used game theory approach to analyze book haram insurgency issues, while [17, 18] used mathematical modeling to analyze insurgency problems in far north of Nigeria.
Despite this growing body of work, existing models often adopt purely ODE frameworks, neglecting the spatial spread of ideology and recruitment, a core feature in real-world

 

insurgency, particularly in regions with porous borders and inaccessible terrain.
This study is motivated by the pressing need to capture the spatiotemporal evolution of insurgency in a more realistic and policy-relevant manner. Unlike traditional compartmental models that assume well-mixed populations, we propose a reaction-diffusion PDE model that incorporates geographical spread**,** nonlinear recruitment**,** relapse dynamics, and security intervention mechanisms. The novelty of this work lies in:

- Modeling relapse and recycling of reformed individuals into insurgency through a dynamic reintegration mechanism.
- Incorporating spatial diffusion to simulate how insurgent ideologies and recruitment strategies spread across geographical regions or hideouts.
- Embedding nonlinear saturating interaction terms to reflect peer influence, cross-sector luring, and ideological indoctrination.
- Implementing a finite difference numerical scheme to simulate the spatial-temporal evolution of all four compartments (Vulnerable, Insurgents, Security Forces, Reformed).
- Providing a comprehensive framework using real variable and parameter estimates that aligns with real conflict dynamics in Nigeria.

This approach bridges the gap between local-scale behavior and macro-scale insurgency spread which offers a robust tool for intervention planning, risk assessment, and counter insurgency strategy evaluation.

## 2. Materials and methods

Insurgency remains one of the most pressing security challenges in Nigeria, with several northern regions severely affected by conflicts involving armed insurgents, local communities, and security forces. Existing models of insurgency often ignore the spatial dynamics of hideouts, camps, and tactical movements, which are crucial for designing localized interventions. This study develops a deterministic, and spatial-temporal mathematical model of insurgency using a nonlinear reaction-diffusion framework.
The total population is divided into four interacting compartments that vary with time *t* and space *x* and later in space *(x, y)*:
The formulation incorporates:

- Diffusion terms to capture random movement or dispersal across geographical space;
- Advection (directed movement) to represent targeted displacement, patrol, or strategic migration of each group;
- Nonlinear reaction terms that govern interactions between groups such as recruitment, relapse, suppression, and reintegration;
- Spatially varying coefficients to reflect the heterogeneity of risk factors, governmental presence, terrain accessibility, socio-political influence, and intervention intensity across locations;
- Saturating functions in interaction terms to ensure realistic bounds in recruitment and conversion dynamics.

The model seeks to provide insight into the effectiveness of spatially distributed interventions, identify recruitment hotspots, and explore long-term patterns of insurgency spread, decline, or stabilization. It also serves as a tool for assessing policy outcomes under varying deployment strategies, disarmament efforts, and community-based reform initiatives.
The system of partial differential equations is solved subject to spatially dependent initial conditions and homogeneous Neumann (no-flux) boundary conditions, ensuring population containment within the considered geographical domain.

**Vulnerable Individuals V (x, t)**

This class includes individuals at risk of being recruited into insurgency due to unemployment, injustice, marginalization, or peer influence. Their dynamics are governed by:

$$\frac{\partial V}{\partial t} = D_V \nabla^2 V - \nabla(V v_V) + \Lambda(\mathrm{x}) - \frac{\beta_1(x) V I}{1+\kappa_1 I} - (\delta + \mu)(x) V \qquad (1)$$

**Explanation of modeling flows and terms in (1)**

- $D_V \nabla^2 V$: Random spatial diffusion (e.g. migration).
- $\nabla(V v_v)$ − Directed movement**:** they might move toward towns, job centers, or flee conflict zones.
- $\Lambda$(x): Inflow of new vulnerable individuals (e.g., due to job loss, school closures).
- $\frac{\beta_1(x) V I}{1+\kappa_1 I}$ : Saturated recruitment into insurgency via interaction with insurgents.
- δ(x)V: Entry into reform programs (NGO work, education, social intervention).
- $\mu_v(x) V$: Natural death rate.

**Active Insurgents I (x, t)**

This group comprises individuals actively participating in armed rebellion. Their growth arises from luring of vulnerable individuals, compromise of security agents, relapse of reformed individuals, peer radicalization, and external support:

$$\frac{\partial I}{\partial t} = D_I \nabla^2 I - \nabla(I v_i) + \frac{\beta_1(x) V I}{1+\kappa_1 I} + \frac{\beta_2(x) V S}{1+\kappa_2 S} - \frac{\beta_3(x) I S}{1+\kappa_3 S} + \eta(x) R + \frac{\omega(x) I^2}{1+\kappa_4 I} + \psi(x) - \big((\mu + \sigma)(x)\big) I - u_I(x) I \qquad (2)$$

**Explanation of modeling flows and terms in (2)**

- $D_I \nabla^2 I$: Random diffusion spread of insurgents across geography.

- $\nabla(Iv_i)$: Tactical movement, often away from security agents.
- $\frac{\beta_1(x)V\,I}{1+\kappa_1 I}$ : Luring or influence from vulnerable population.
- $\frac{\beta_2(x)V\,S}{1+\kappa_2 S}$: Defection of security forces corrupting vulnerable individuals.
- $\frac{\beta_3(x)I\,S}{1+\kappa_3 S}$: Reformation or arrest of insurgents by security forces.
- $\eta(x)R$: Relapse of reformed individuals.
- $\frac{\omega(x)I^2}{1+\kappa_4 I}$: Peer radicalization, e.g., self-organized cells.
- $\psi(x)$: External support, e.g., weapons, logistics.
- $\mu I$: Natural death.
- σ(x): Elimination via military action.
- $u_I(x)I$ : Disarmament efforts (buy-back, amnesty, drone strikes).

**Security Forces S (x, t)**

This group represents the government or organized defense systems deployed to combat insurgents:

$$\frac{\partial S}{\partial t} = D_S\nabla^2 S - \nabla(Sv_s) + \alpha(x) - (\gamma+\mu)(x)S - \frac{\beta_3(x)I\,S}{1+\kappa_3 S} - u_S(x)S \qquad (3)$$

Explanation of modeling flows and terms in (3)

- $D_S\nabla^2 S$: Random patrol-based movement.
- $\nabla(Sv_s)$: Directed deployment e.g., toward hotspots.
- α(x): Planned deployment by government.
- γ(x) S: Attrition due to fatigue, injury, or desertion.
- $\mu(x)$: Natural death rate
- $u_S(x,t)$ Corruption or inefficiency, e.g., local administrative failure.

**Reformed Individuals R (x, t)**

These are individuals who transitioned out of insurgency through de-radicalization or public reform programs:

$$\frac{\partial R}{\partial t} = D_R\nabla^2 R - \nabla(Rv_R) + \frac{\beta_3(x)I\,S}{1+\kappa_3 S} + \delta(x)V - (\mu+\eta)(x)R - u_R(x)R. \qquad (4)$$

**Explanation of modeling flows and terms in (4)**

- $D_R\nabla^2 R$ : Spatial diffusion (e.g., relocation, resettlement).
- $\nabla(Rv_R)$ : Movement toward civil zones**,** NGO programs, or schools.

$\frac{\beta_3(x)I\,S}{1+\kappa_3 S}$: Reformed insurgents via security pressure.
δ(x): Reformed vulnerable individuals via social effort.
η(x): Relapse due to failed reintegration or societal rejection.
$u_R(x,t)R$: Policy failure, underfunded reform, stigma.

The full system of reaction-diffusion equations is given as:

$$\frac{\partial V}{\partial t} = D_V\nabla^2 V - \nabla(Vv_V) + \Lambda(x) - \frac{\beta_1(x)V\,I}{1+\kappa_1 I} - (\delta+\mu)(x)V,$$
$$\frac{\partial I}{\partial t} = D_I\nabla^2 I - \nabla(Iv_i) + \frac{\beta_1(x)V\,I}{1+\kappa_1 I} + \frac{\beta_2(x)V\,S}{1+\kappa_2 S} - \frac{\beta_3(x)I\,S}{1+\kappa_3 S} + \eta(x)R + \frac{\omega(x)I^2}{1+\kappa_4 I} + \psi(x) - \left((\mu+\sigma)(x)\right)I - u_I(x)I \qquad (5)$$
$$\frac{\partial S}{\partial t} = D_S\nabla^2 S - \nabla(Sv_s) + \alpha(x) - (\gamma+\mu)(x)S - \frac{\beta_2(x)V\,S}{1+\kappa_2 S} - u_S(x)S$$
$$\frac{\partial R}{\partial t} = D_R\nabla^2 R - \nabla(Rv_R) + \frac{\beta_3(x)I\,S}{1+\kappa_3 S} + \delta(x)V - (\mu+\eta)(x)R - u_R(x)R.$$

Subject to initial Conditions**:**

$$V_o(x,0) = V_o(x), I_o(x,0) = I_o(x), S_o(x,0) = S_o(x), R_o(x,0) = R_o(x) \qquad (6)$$

And boundary Conditions (No-Flux)**:**

$$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial t} = 0, \text{for } x = (0, L) \qquad (7)$$

Model Assumptions guiding (5)-(7) are as Follows:
- The population is closed within a bounded spatial domain x ∈ [0, L].
- Recruitment into insurgency is driven by interaction between insurgents and vulnerable individuals.
- A portion of security personnel may become insurgents due to ideological persuasion or fear.
- Reform and relapse are possible via structured programs and social influence, respectively.
- Individuals can diffuse through space, and this diffusion follows Fick's law.
- Government policies (recruitment, reform, awareness) are implemented uniformly over the domain. To analyze the spatial and temporal dynamics of insurgency and its control mechanisms, we adopt a reaction-diffusion framework involving four compartments: vulnerable individuals $V$, active insurgents $I$, security forces $S$, and reformed individuals $R$. Two different computational scenarios are considered: one in a single spatial direction $(1D)$ and the other in two spatial dimensions $(2D)$. Both implementations are carried out in Python using the FDM combined with the MOL and time integration via SciPy's *solve_ivp* solver.

**One Dimensional Spatial Simulation**

In the first approach, we consider insurgency dynamics evolving along a single spatial axis, such as a highway or conflict-prone stretch of land.

- Spatial Discretization**:** The domain is defined on $x \in [0, 100]km$ and discretized uniformly into $N_x =$

$100N$ grid points, giving a spatial resolution of Δ x=1 km.

- Finite Difference Scheme: The diffusion terms (second-order derivatives) in each PDE are approximated using the standard central difference scheme:

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1}-2u_i+u_{i-1}}{\Delta x^2} \quad (8)$$

Zero-flux (Neumann) boundary conditions are enforced by padding the edge values during Laplacian computation to ensure no population flow into or out of the spatial domain.

- Initial Conditions**:** To capture realistic localized phenomena:
  - $V_o(x)$: Gaussian profile centered at x = 50, simulating an at-risk population core.
  - $I_o(x)$: Initiated near x = 30, representing an insurgency hotspot.
  - $S_o(x)$: Peaked at x = 70, modeling a security base or garrison.
  - $R_o(x)$ : Set to zero, allowing reformed individuals to emerge through system dynamics.

- Time Integration: The method of lines (MOL) is applied by flattening each compartment's spatial profile into a 1D vector. The resulting system of ODEs is integrated from $t = 0 - 100t$ days using the adaptive 4th/5th-order Runge-Kutta method (RK45) via SciPy's solve_ivp, with 100 time t for solution tracking.

**Two-Dimensional (2D) Spatial Simulation.**

To reflect more realistic spread patterns of insurgency across geographical regions, the second simulation extends the model to two spatial directions, *x* and *y*, capturing urban centers, rural corridors, and conflict diffusion dynamics.

- **Domain and Discretization**: A square domain of 50×50 km² is discretized into a uniform 30×30 grid. The grid spacing is Δ x = Δ y ≈ 1.72 km.
- **Laplacian Approximation**: The two-dimensional Laplacian is approximated using a 5-point stencil finite difference scheme:

$$\nabla^2 U_{i,\ j} \approx \frac{U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} - 4U_{i,j}}{\Delta x^2}. \quad (9)$$

Neumann boundary conditions are applied via edge-padding to preserve population continuity at the borders.

- **Initialization**:
  - Vulnerable individuals $V_o(x,y)$: Centered at (25, 25) km.
  - Insurgents $I_o(x,y)$: Localized at (15, 15) km.
  - Security forces $S_o(x,y)$: Focused at (35, 35) km.
  - Reformed individuals $R_o(x,y)$ : Zero-initialized.
- **Numerical Integration**: Using the MOL strategy, each population matrix is flattened to a vector, and the coupled system is integrated using solve_ivp over 20 days. The final solutions are reshaped and visualized as 3D surface plots to reveal the geographic distribution of each compartment at the simulation endpoint.

The parameter values used in both simulations are informed by literature, security reports, and assumptions grounded in the context of insurgency-prone regions in sub-Saharan Africa, particularly northern Nigeria:

**Table 1. Variable and Parameter Descriptions and Values**

| Symbol | Description | Value/Unit | Source/Assumption |
|---|---|---|---|
| V (x, t) | Vulnerable individuals | 0.05 km²/day /persons | Limited passive movement (Assumed) |
| I (x, t) | Active insurgents | 0.08 km²/day /persons | High mobility of insurgents (Assumed) |
| S (x, t) | Security forces | 0.06 km²/day /persons | Active patrolling (Assumed) |
| R (x, t) | Reformed individuals | 0.02 km²/day /persons | Minimal movement post reform (Assumed) |
| $D_V$ | Diffusion rate of vulnerable group | km²/day | Assumed small (Assumed) |
| $D_I$ | Diffusion rate of insurgents | km²/day | Mobility-based (Assumed) |
| $D_S$ | Diffusion rate of security forces | km²/day | Patrol-based (Assumed) |
| $D_R$ | Diffusion rate of reformed individuals | km²/day | Low |

| | | | |
|---|---|---|---|
| | | | (Assumed) |
| Λ(x) | Inflow rate into vulnerable which may be due to job loss or lack of education | 5 - 10 persons/day | [5-7, 10, 13] |
| $\beta_1(x)$ | Luring rate of vulnerable by insurgents into insurgency | 0.022 person/day | [5-7, 10, 13] |
| $\beta_2(x)$ | Security compromise rate luring vulnerable individuals into crime of insurgency | 0.03 person/day | [5-7, 10, 13] |
| $\beta_3(x)$ | Contact rate leading to Reformation or arrest of insurgents by security forces | 0.24 person/day | [5-7, 10, 13] |
| η(x) | Relapse rate of reformed individuals | 0.14 person/day | Assumed |
| δ(x) | Entry into reform programs for enlightenment against becoming an insurgent | 0.04 person/day | Estimated |
| μ(x) | Natural death rate | 0.1 person/day | [5-7, 10, 13] |
| α(x) | Deployment rate of security personnel | 0.001 person/day | [5-7, 10, 13] |
| γ(x) | Attrition rate due to fatigue, injury, desertion or killings by insurgents | 0.22 person/day | Assumed |
| σ(x) | Elimination of insurgents by military action | 0.16 person/day | [17, 18] |
| ω(x) | Peer influence or radicalization among insurgents | 0.33 person/day | Assumed |
| ψ(x) | External support to insurgents | 0.17 person/day | Assumed |
| $k_1$ | Saturation constant | 0.23 person/day | Assumed |
| $k_2$ | Saturation constant | 0.21 person/day | Assumed |
| $k_3$ | Saturation constant | 0.22 person/day | Assumed |
| $k_4$ | Saturation constant | 0.24 person/day | Assumed |
| $u_I$ | Disarmament rate | 0.14 person/day | Assumed |
| $u_S$ | Corruption or inefficiency rate | 0.11 person/day | Assumed |
| $u_R$ | Policy failure or underfunded reform rate | 0.13 person/day | Assumed |

**3. Results and Discussion:**

After numerical integration, the model state variables were extracted at the final simulation time and plotted as spatial distributions. These plots revealed the emergence of insurgency hotspots, spread of vulnerability, deployment of security, and localized success of reform programs.

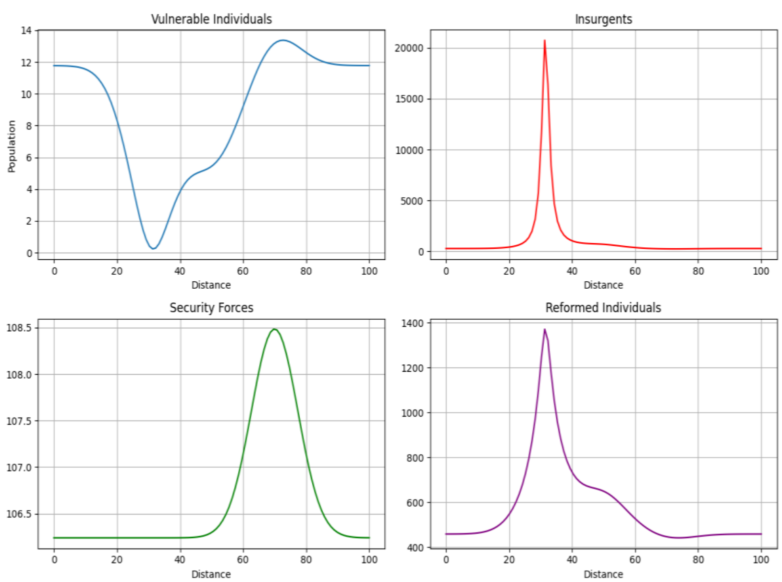


Figure 1. One dimensional spatial distribution of population compartments: (a) Vulnerable individuals V(x, t), (b) Insurgents I(x, t), (c) Security forces S(x, t) and (d) Reformed individuals R(x, t) at t = 50 days across the domain x∈[0,100] km.

In view of the one-dimensional spatial domain, Figure 1 presents the simulation results of the proposed spatiotemporal insurgency model at the final time point t = 100 days, showing the dynamics of each compartment across the one-dimensional spatial domain.

The first upper left sub-figure illustrates the distribution of vulnerable individuals V (x, t), where there is a significant depression in population density between x = 25 km and x = 50 km. This decline shows regions where intense recruitment into insurgency has occurred due to high insurgent presence. On the other hand, elevated values at the domain edges reflect lower exposure and thus sustained vulnerability.

The upper left sub-figure displays the profile of active insurgents I (x, t), revealing a sharp peak around x = 30 km, with population levels exceeding 20,000. This concentration reflects the epicenter of insurgent activity, sustained by successful recruitment of vulnerable individuals, compromise of security personnel, relapse of reformed individuals, and external support mechanisms. The sharpness of the peak suggests a highly localized but powerful insurgent presence.

The lower left sub-figure shows the spatial distribution of security forces S (x, t), which peaks around x=70 km. The spatial offset between the insurgent and security force peaks suggests a suboptimal deployment strategy or delayed response. The relatively flat gradient in other regions indicates limited mobility and diffusion, which may hinder effective engagement with insurgent strongholds.

The lower left sub-figure depicts the evolution of the reformed population R (x, t) peaking near the insurgent-dense region at (x) =30 km. This alignment reflects successful de-radicalization efforts where insurgency is most severe. However, the magnitude of this peak is significantly lower than that of insurgents, indicating that reform interventions, while functional, are yet insufficient to counteract the scale of radicalization.

Important observations as regards Figure 1.

- Localized Dynamics**:**Insurgency remains highly concentrated in specific zones, validating the use of spatially explicit modeling to capture asymmetric conflict behavior.
- Security Mismatch**:** There is an evident spatial lag in security force presence relative to insurgent hotspots, which may reduce the effectiveness of counterinsurgency efforts.
- Policy Implications**:** Strategic redeployment of security forces toward high-insurgency zones, along with increased reform and preventive outreach in vulnerable regions, could significantly suppress insurgent growth.
- Model Validation**:** The results align with real data estimation observations in regions affected by insurgency, such as northern Nigeria and parts of the Middle Belt, where insurgent activity tends to cluster and exploit socio-economic vulnerabilities.

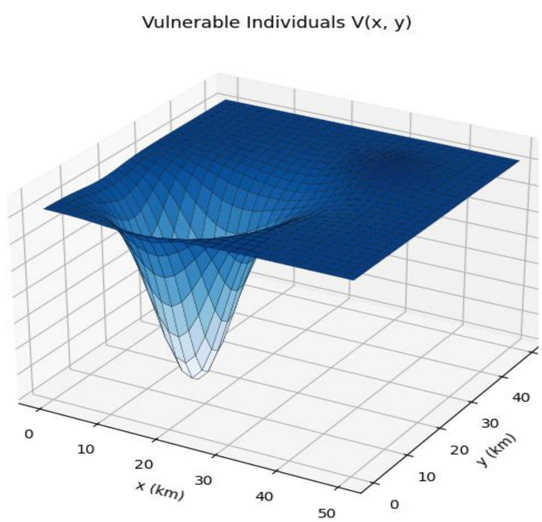


**Figure 2**: 2 dimensional spatial distributions of the vulnerable model compartment $V$ at time t = 50 days across a 100 km domain.

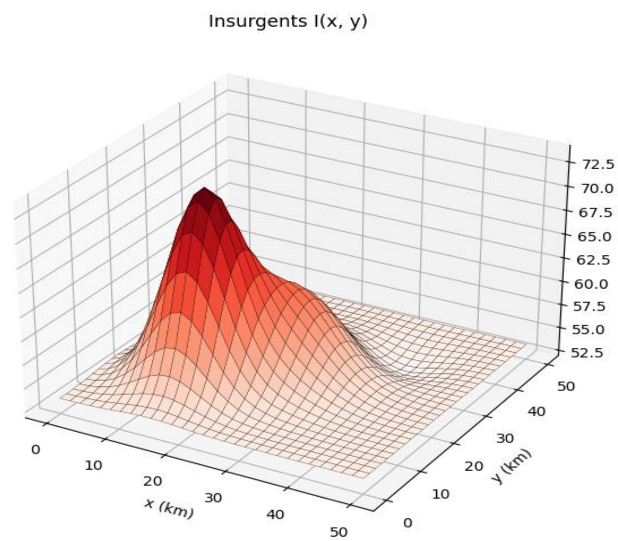


**Figure 3**: 2 dimensional spatial distributions of insurgent model compartment $I$ at time t = 50 days across a 100 km domain.

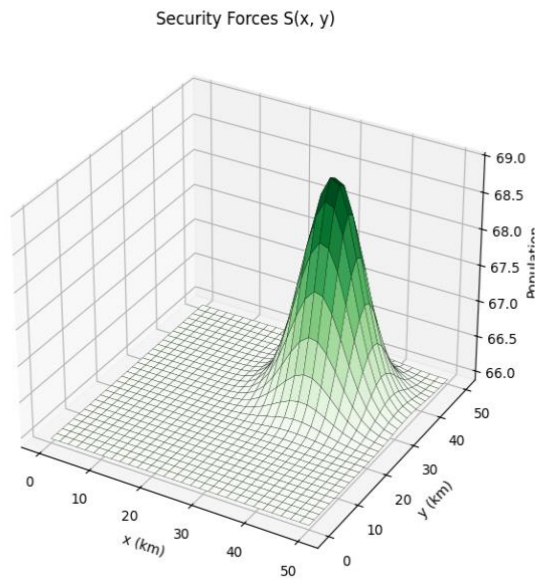


**Figure 4**: 2 dimensional spatial distributions of security forces S model compartment at time t = 50 days across a 100 km domain.

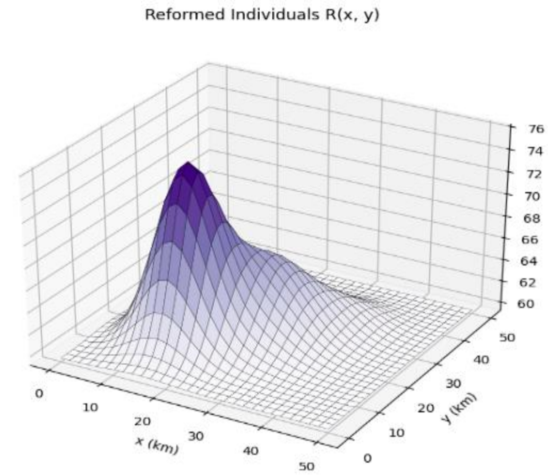


**Figure 5**: 2 dimensional spatial distributions of reformed model compartment R at time t = 50 days across a 100 km domain.

In view of the 2-dimensional domain analysis, Figures 2, 3, 4 and 5 displays the final spatial distributions of insurgency model compartments at time t = 50 days across a 100 km domain. Figure 2, displays the profile of vulnerable individuals $V(x, y, t)$, which peaks around the mid-point (x, y) = 50 km, showing the initial hotspot of socio-economic vulnerability. Over time, vulnerability has slightly diffused outward but remains concentrated by indicating limited spread due to moderate diffusion $D_V = 0.05$ and ongoing recruitment into insurgency or reform. The effect of luring $\beta_1(x, y)$ and empowerment $\delta(x, y)$ appears significant, but not enough to exhaust the vulnerable pool.

Figure 3 shows the insurgent population $I(x, y, t)$, initially located at $(x, y) = 30$ km has propagated and grown to form a broad hump across the region. This behavior stems from sustained recruitment from both vulnerable and compromised security agents $\beta_1(x, y)$, $\beta_2(x, y)$ as well as self-radicalization $\omega$ and external support $\psi(x, y)$. However, the population declines at the boundaries due to security suppression and natural decay $\mu(x, y)$, and $\sigma(x, y)$. The plot reveals multiple influence zones rather than a single concentrated cell.

Figure 4 shows that the security force distribution $S(x, y, t)$ remains relatively stable around the initial location $(x, y) = 70$ km, with some mild spatial dispersion due to patrol diffusion $D_S$. The slow attrition rate $\gamma(x, y)$ and constant recruitment $\alpha(x, y)$ to ensure their presence is maintained. However, the failure to significantly penetrate insurgent regions suggests either weak spatial deployment or overwhelming opposition in those zones.

Finally, Figure 5, showed that, the reformed class $R(x, y, t)$ which started at zero, gradually builds up across the domain, with visible peaks where insurgents and vulnerable populations overlap. This confirms that effective reform policies $\beta_3(x, y)$ and empowerment $\delta(x, y)$ are driving transitions from conflict to peace. The presence of relapse η(x, y) slightly offsets this trend but does not dominate.

## 4. Conclusion

This study presents a novel spatiotemporal reaction-diffusion model for analyzing the dynamics of insurgency in conflict-prone regions in Nigeria. By incorporating four critical compartments namely vulnerable individuals, active insurgents, security forces, and reformed individuals. The model illustrates the intricate interplay between socio-political vulnerabilities, recruitment, spatial diffusion, and policy interventions. Through a combination of one-dimensional and two-dimensional simulations using FDM and time integration via the MOL, the framework successfully reveals how insurgency evolves both temporally and spatially.

The results from the 1D simulation reveal a highly localized insurgent buildup around the mid-domain ((x, y) ≈ 30 km), heavily sustained by vulnerable populations, peer influence, and external support. The corresponding lag in the deployment of security forces, peaking instead at (x, y) ≈ 70 km, underscores a critical spatial mismatch that weakens the effectiveness of counterinsurgency strategies. While the reformed class emerges near the insurgent core, its magnitude remains inadequate to counterbalance the radicalization momentum.

The 2-dimensional model further demonstrates how insurgency diffuses across both spatial directions, generating regional conflict cells influenced by the initial positioning of vulnerabilities and security response. Vulnerable and insurgent populations tend to cluster in overlapping regions, while security presence often trails behind or remains static, limiting its suppressive capacity. Reform interventions, though positive, appear spatially constrained and partially reactive rather than preventive.

From these findings, several key conclusions emerge:

- Spatial Heterogeneity Matters: Insurgency is not uniformly distributed; rather, it forms clustered hotspots. Therefore, further models must go beyond temporal averages to include spatial variability in both populations and interventions.
- Insufficient Security Penetration**:** Despite ongoing recruitment, the spatial positioning of security forces does not effectively overlap with insurgent hotspots. This diminishes counterinsurgency impact and highlights the need for geographically informed redeployment strategies.
- Reactive Reform Is Inadequate**:** Although reform efforts yield results, they remain spatially and quantitatively insufficient. Reliance on post-insurgency reform without adequate pre-emptive empowerment may fail to neutralize radicalization cycles.
- Systemic Feedback Loops Are Crucial**:** Nonlinear interactions such as relapse, self-radicalization, and recruitment from compromised security agents reinforce the persistence of insurgency. These dynamics must be accounted for in any long-term stabilization strategy.

The following recommendations are posed to policy makers.

- Geographically Targeted Security Deployment**:** Security resources should be dynamically allocated based on spatial intelligence to ensure that presence aligns with insurgent concentrations. The current static or lagging deployment weakens operational outcomes.
- Community-Based Prevention Programs**:** Vulnerable hotspots identified by the model (e.g., the central domain) require intensive socio-economic support, job creation, education, and grievance redress to reduce recruitment potential.
- Decentralized Reform Centers: Rehabilitation and reintegration efforts should be spatially dispersed and closer to insurgent hubs to effectively convert active members and reduce relapse.
- Integration of Real-Time Spatial Data: Policy design should be supported by geospatial insurgency tracking and modeling to update intervention zones dynamically, rather than relying solely on static strategies.
- Further Model Extensions**:** Future work should incorporate more detailed factors such as terrain, population density, displacement dynamics, and stochastic effects to capture real-world irregularities and improve forecasting.

## CRediT authorship contribution statement:

Oluwatayo Michael Ogunmiloro contribute to the conceptualization, methodology, software application, validation, and formal analysis of the work. Akinyele Yussuff Akinola performed simulation, review and editing of the work. All authors have read and agreed to the published version of the manuscript.

## Data availability statement

No real data was used for the work.

## Declaration of competing interest

The authors declare that they have no known competing interests.

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