The impact of the suction process on the blood flow in the peristaltic horizontal vein of Henle (Gastro-colic venous trunk GCT)

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Abstract: The blood flow in horizontal veins is studied before without consideration of gravity term and constant pressure gradient. This study examines the effects of suction and gravity on the blood flow in the peristaltic horizontal vein of Henle (GCT, gastro-colic venous trunk). The blood flow in the peristaltic horizontal vein of Henle (GCT) is considered as an incompressible and viscous Newtonian fluid properties. Equations for Navier-Stokes, concentration, volume rate, and mass conservation define the mathematical model of the current problem. When blood has long blood wavelengths and a low Reynolds number, the analytical solutions are obtained under the influence of the amplitude ratio, blood diffusion coefficient, Grashof number, and suction process parameter. The pressure gradient, the blood velocity, the blood concentration, and the stream function of the blood flow are obtained. The blood concentration proportional inversely with suction process. On contrary the blood velocity increases under the effect of suction process. The results are shown in several graphs for different values of the physical parameters. The majority of diseases need to the suction process as a wet and dry cupping therapy treatments to avoid the blood clots appeared. **Keywords:** Flow of blood; Gastro-colic venous trunk GCT; Suction process; Pressure gradient; Peristaltic motion.

1. Introduction

The meaning of some physical contracts are present in physics, chemistry, and medicine. The hot-cold, absorptionemission, water source -water sink, heating source-heating sink, suction-injection [**1**]. The contracts in suction and injection processes are play a real contribution in medicine. Injection process is very important for curing many diseases [**2**]. On contrary, suction process as a wet and dry cupping therapy treatment are effective for many diseases such as pains in mmuscle pain, cervical and lumbar vertebrae, bio tissues and covid-19 symptoms [**3-4**]. The colon function is very important for any human. In the follows, the contents of veins in colon and relations between them are illustrated.

The ascending colon, transverse colon, descending colon, and sigmoid colon make up the human colon. The inferior and superior mesenteric veins receive the colic venous drainage [**5- 8**]. The right colic vein drains the ascending colon, while the center colic vein drains the transverse colon. The vein of Henle, first described in 1868, is the term used to describe the gastrocolic trunk. As depicted in Fig. 1, it is also known as the gastrocolic trunk of Henle (GTH). In surgical operations for the colon, pancreas, and stomach, understanding the Henle vein is helpful [**5**].

One frequent variation in the anatomy of portal flow is the vein of Henle. Although it is a very diverse channel, the gastropancreatic-colic trunk is the most common kind. Henle's vein has a diameter. Its 4.2 mm mean diameter. Its length of 10.7 mm was only documented in one study [**8**]. It is where the right superior colic and the right gastroepiploic veins meet. At the pancreas' neck, it empties blood into the superior mesenteric vein [**6**]. The vertical inferior mesenteric vein's peristaltic flow

for a mixture of blood and gas bubbles is investigated [**9,11- 12**].

Fig.1. Sketch of gastrocolic trunk vein

Maha et al. have obtained analytical solutions for a nonlinear system of equations that explain the peristaltic motion of particular veins under the influence on particular physical parameters [**7,13-14**].

The Henle horizontal peristaltic vein's suction process parameter served as the blood flow's primary determining factor (GCT, gastro-colic venous trunk) [**9,15-16**]. The peristaltic fluid flow in vertical cylindrical tubes is studied by many authors [**17-19**]. Recently, the modeling of bio fluids flow in a human colon is studied in details by Sara Elkholy [**24**]. A numerical solution of 2-D single-phase-lag (SPL) bioheat model using alternating direction implicit (ADI) finit

difference methoied by Ibrahim Abbas et. A. [**10,13,20**] is stude. The blood flow in the peristaltic horizontal vein of Henle (gastrocolic venous trunk) is considered as an incompressible and viscous Newtonian fluid properties. Recently the blood flow is studied in a vertical vein in detail by Elbendary et al [**9,12**]. The peristaltic fluid flow is described by linear Burger and Navier-stokes equations [**14,21-23**].

Nomenclatures

In this paper, the blood flow in the gastro-colic venous trunk (GCT), as horizontal peristaltic vein of Henle is studied. The gravitational field is regarded as a physical variable. The mathematical model is based on Navier-Stokes, concentration, and volume rate equations, The concentration of blood, velocity, pressure gradient equations and stream function are obtained as a solution of proposed model. Moreover, the obtained solutions are valid only for low Reynolds numbers and large wavelengths($\delta \rightarrow 0$). The concentration and blood velocity are affected by the suction processes.

2. Materials and methods

According to a mathematical simulation of a current problem, blood travels down the peristaltic wall of the horizontal vein of Henle in a cylindrical coordinate (r, z) with z measured along the vein's axis and r being a radius in the radial direction as shown in Fig. 2.

This sinusoidal wave has a small amplitude. The following equation provides the vein's border.

$$
H(\bar{Z},\bar{t}) = a + b\sin(\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t})).\tag{1}
$$

Introducing a wave frame (\bar{r}, \bar{z}) moving with velocity c away from the fixed frame $(\overline{R}, \overline{Z})$, the transformations

$$
\begin{aligned}\n\bar{r} &= \bar{R}, \quad \bar{z} = \bar{Z} - c\,\bar{t} \quad \bar{u} = \bar{U} - c, \ \bar{w} = \bar{W}, \\
h &= 1 + e\sin(2\pi z),\n\end{aligned} \tag{2}
$$

where (\bar{u}, \bar{w}) are the velocity components in the moving frame.

Fig. 2. Sketch of model

The mathematical model of the physical problem in the wave frame is described by mass, Navier-Stokes, concentration and volume rate equations as follows:

Mass equation
\n
$$
\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{u}) + \frac{\partial \bar{w}}{\partial \bar{z}} = 0.
$$
\n(3)
\nNavier-Stokes equations
\n
$$
\rho_b \left(\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} \right) = -\frac{\partial \bar{P}}{\partial \bar{r}} + \eta \left\{ \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{u}}{\partial \bar{r}} \right) - \frac{\bar{u} + c}{\bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}} \right\},
$$
\n(4)
\n
$$
\rho_b \left(\bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} \right) = -\frac{\partial \bar{P}}{\partial \bar{z}} + \eta \left\{ \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{w}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right\} +
$$
\n
$$
\rho g \alpha (\bar{C} - C_0) \sin \left(\frac{2\pi}{\lambda} \right) \bar{z}.
$$
\nConcentration equation
\n(5)

$$
\bar{u}\frac{\partial \bar{c}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{c}}{\partial \bar{z}} = D_T \left(\frac{\partial^2 \bar{c}}{\partial \bar{r}^2} + \frac{1}{r}\frac{\partial \bar{c}}{\partial \bar{r}} + \frac{\partial^2 \bar{c}}{\partial \bar{z}^2}\right) + \frac{b}{r}\frac{\partial \bar{c}}{\partial \bar{r}}.
$$
\nVolume flow rate equation

$$
\bar{q}_b = 2 \int_0^h \bar{r} \ \overline{w} \ (\bar{r}, \bar{z}) d\bar{r} \ , \tag{7}
$$

where P is indicated to the pressure, η is the viscosity of blood, C is indicated to the concentration of blood in the horizontal vein of Henle, q is the gravitational acceleration, α is the concentration coefficient of volumetric expansion, and ρ is indicated to the density of blood. The relation between the dimensional and non-dimensional variables has the form

$$
\begin{aligned}\n\bar{r} &= a \, r \,, \quad \bar{z} = \lambda \, z \,, \qquad \bar{u} = c \delta \,, \quad w = c \, \bar{w} \,, \quad \bar{q} = c a \, q \,, \\
\delta &= \frac{a}{\lambda} \,, \quad \bar{t} = \frac{\lambda}{c} t \,, \qquad H = \frac{h}{a} \,, \qquad e = \frac{b^*}{a} \,, \qquad G_c = \frac{\rho g \alpha a^2 C_0}{\eta c} \,, \quad p = \frac{a^2}{c \lambda \eta} \,\bar{p} \,, \\
\Phi &= \frac{c - c_0}{c_0} \,, \quad R_e = \frac{\rho c a}{\eta} \,,\n\end{aligned} \tag{8}
$$

where, δ is the wave number, R_e is indicated to the Reynolds number, e is the amplitude ratio, G_c is indicated to the modified Grashof number and Φ is indicated to the blood concentration. Substituting from relations (8) into the equations (3-7), then

$$
\frac{1}{r}\frac{\partial}{\partial r}(r u) + \frac{\partial w}{\partial z} = 0,
$$
\n
$$
R_e \delta^3 \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial r} + \delta^2 \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) - \frac{u+1}{r^2} + \delta^2 \frac{\partial^2 u}{\partial z^2} \right\},
$$
\n(9)

(10)

$$
R_e \delta \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \delta^2 \frac{\partial^2 w}{\partial z^2} + G_c \Phi \sin (2\pi z) (11)
$$

$$
c a \delta \left(u \frac{\partial \Phi}{\partial r} + w \frac{\partial \Phi}{\partial z} \right) = D_T \left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \left(1 + \frac{b}{D_T} \right) \frac{\partial \Phi}{\partial r} + \delta^2 \frac{\partial^2 \Phi}{\partial z^2} \right),\tag{12}
$$

$$
q_b = 2 \int_0^h r w(r, z) dr, \qquad (13)
$$

where Φ_0 indicated to initial blood concentration at the wall of vein. The above system (9-13) is called the non-dimensional form. The physical problem is solved for the long wavelength $(\delta \ll 1)$ and the Reynolds number is quite small $(R_e \rightarrow 0)$, then the equations (10-12) become

$$
\frac{\partial P}{\partial r} = 0, \qquad (14)
$$
\n
$$
\frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + G_c \Phi \sin(2\pi z), \qquad (15)
$$

$$
\frac{\partial z}{\partial r^2} + \frac{1}{r} \left(1 + \frac{b}{p_T} \right) \frac{\partial \Phi}{\partial r} = 0.
$$
 (16)

In the following, the flow of blood in a horizontal peristaltic vein of Henle (Gastro-colic venous trunk (GCT)) is formulated by equations (14-16) in terms of pressure, blood velocity, and concentration. The equation (16) under the following dimensionless boundary conditions

at
$$
r = 0, \qquad \Phi = \Phi_1, \qquad (17)
$$

and
$$
r = h
$$
, $\Phi = \Phi_0$, (18)

is solved and the concentration equation takes the form

$$
\Phi(r, z) = \Phi_1 + \frac{c_1 p_T}{b} r^{\frac{-b}{D_T}}
$$
\n(19) where,

$$
C_1 = \frac{b(\Phi_1 - \Phi_0)}{p_T} h^{\frac{b}{D_T}}, \quad \Phi_1 - \Phi_0 > 0.
$$
 (20)

Substituting from Eq. (19) in Eqn. (15) under the following boundary conditions

at
$$
r = 0
$$
, $\frac{\partial w}{\partial r} = 0$, (21)

and at $r = h$, $w = w_0$. (22) and then Eqn. (15) is solved in terms of the following blood

velocity

$$
w(r, z) = w_0 + \frac{(r^2 - h^2)(\frac{dP}{dz})}{4} - G_c \sin(2\pi z) \left(\frac{\Phi_1(r^2 - h^2)}{4} - \frac{(\Phi_1 - \Phi_0)}{\left(2 - \frac{b}{2r}\right)^2} \left(h^{\frac{b}{D_T}} r^{2 - \frac{b}{D_T}} - h^2 \right) \right).
$$
 (23)

On the basis of the above blood velocity and volume rate equations (23) and (13) respectively, the pressure gradient of blood flow takes the form

$$
\frac{dP}{dz} = \frac{8 w_0}{h^2} - \frac{8q}{h^4} + G_C \left(\Phi_1 + \frac{8 (\Phi_1 - \Phi_0) \left(\frac{b}{D_T} - 2\right)}{\left(4 - \frac{b}{D_T}\right) \left(2 - \frac{b}{D_T}\right)^2} \right) \sin(2\pi z). \quad (24)
$$

The stream function can be obtained from blood velocity and continuity equations (23) and (9) respectively in the form

$$
\Psi(r, z) = \frac{1}{16}r^2 \left(8A_0 + (-2h^2 + r^2) \left(\frac{dP}{dz}\right) + \frac{2h^2 \sin[360z]G_C(b(-4a+b)\phi_0 + 4a^2\phi_1)}{(b-2a)^2}\right).
$$
\n(25)

3. Results and Discussion:

The mass, Navier-Stokes, volume rate, and concentration equations (3–7), for an incompressible and viscous Newtonian blood flow through the peristaltic horizontal vein of Henle

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(gastrocolic venous trunk) under the influence of suction process and gravity term, respectively, describe the mathematical model of the current problem. Analytical solutions are used to determine the concentration, blood velocity, pressure gradient, and stream function under the influence of a number of physical parameters, including the suction parameter b, Grashof number G_C , blood diffusion coefficient D_T and amplitude ratio e. The non-linear system (3-7) is transformed to another non dimensional system (9-13) with the existence of non-dimensional numbers like Grashof and Reynolds numbers. The system (14-16) is obtained for the long wavelength ($\delta \rightarrow 0$). The blood concentration is obtained by relation (19). The blood flow velocity is given by equation (23). The gradient pressure of blood is given by eqns. (24). Finally, the stream function of blood is defined as in equation (25). The numerical values of some physical parameters are $\rho_g = 1.37 ((kg)m^{-3}), \rho_b = 1060 (kg)m^{-3}, \; \Re =$

8.3144 *(mol. K)*,
$$
T = 37K
$$
, $D_T = 2 * 10^{-8} (m^3/$

).Moreover, the variable data are collected in the Table 1.in the form:

Table 1. The variable data

Symbol		r_{vein}	z	A_0	Φ_0	Φ,
Value	0.32	1 mm	0.1			

The blood concentration "Φ" in terms of "r" for varies values of suction parameter "b", blood diffusion coefficient and amplitude ratio is plotted in Figs.3. It is noted that, the concentration of blood is proportional with blood diffusion coefficient and amplitude ratio and inversely with suction parameter.

The blood velocity "w" in terms of "r" for varies values of suction parameter "b", blood diffusion coefficient, amplitude ratio e , and Grashof number is shown in Fig.4. Figure $4(a)$ displays that the velocity profile gains a higher value with increasing b. This means the suction process improves the blood velocity in bio-tissues of veins. Figure 4(b) shows that there is a decline in velocity with increasing blood diffusion coefficient D_T . In Figure. 4(c) It is noted that, the velocity of blood is proportional with amplitude ratio e in the first part of parameter "r" .On contrary, and in the second part of same parameter "r", the relationship is reflected with the same parameters. The velocity profile gains a higher value with increasing G_c as shown in figure 4(d). This means the increasing of blood viscosity tends to decline the blood velocity in bio-tissues of veins. In Figure. 4(e), it is observed that, the existence of suction term improves the blood velocity in bio-tissues of veins.

The gradient pressure of blood " $\frac{dP}{dx}$ $\frac{dr}{dz}$ " in terms of "r" for different values of suction parameter "b", amplitude ratio , and Grashof number is shown in Figs.5. It is noted that, the gradient pressure is proportional with suction parameter but inversely with amplitude ratio, and Grashof number in the bottom of wave. On contrary, at the top of wave, the relationship is reflected with the same parameters.

The Stream function of blood "Ψ" versus the diameter "r" for

different values of "b" is shown in Fig.6. It is noted that the turbulent waves appeared when $b=0$. On contrary, when $b=1$, the stream waves is transformed to weak turbulent flow.

The blood concentration "Φ" in terms of "r" for a limited value of suction parameter "b", is compared with zero suction parameter is plotted in Fig.3(d) It is noted that, the concentration of blood performs lower values under the effect of suction parameter. On contrary, the blood velocity for a limited value of suction parameter "b", is compared with zero suction parameter is plotted in Fig.4(e) It is noted that, the blood velocity performs higher values in the beginning of suction process.

Fig. 3. Concentration distribution Φ versus the diameter r for different values of (a) suction parameter "b", (b) blood diffusion coefficient, (c) amplitude ratio, (d) comparison with zero suction term

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Fig. 4. Blood velocity " w " versus the diameter r for different values of (a) suction parameter "b", (b) blood diffusion coefficient D_T , (c) amplitude ratio e , (d) Grashof number, and (e) comparison with zero suction term

Fig. 5. Pressure gradient of blood " $\frac{dP}{dx}$ $\frac{dr}{dz}$ " versus z for different values of (a) suction parameter "b", (b) amplitude ratio e , and (c) Grashof number

Fig. 6. Stream function of blood flow "Ψ" for various values of suction parameter b where (a) $b=0$ and (b) $b=1$.

4. Conclusion

The flow of blood in a peristaltic horizontal vein of Henle (gastrocolic venous trunk) is described by the system (14-16) when the wavelength is a long one ($\delta \rightarrow 0$). The obtained analytical solutions in terms of blood concentration, velocity, gradient pressure and stream function equations are given by (19), (23), (24) and (25) respectively. The discussion of results and figures concluded the following remarks:

- 1. The concentration of blood is increasing with blood diffusion coefficient and amplitude ratio and decreasing with suction parameter.
- 2. The blood velocity is increasing with amplitude ratio , Grashof number, and suction parameter, but it decreases with blood diffusion. On contrary, in the second part of same parameter "r", the relationship is reflected with the same parameters.
- 3. The pressure gradient of blood is increasing with amplitude ratio, Grashof number and inversely with suction parameter at top of wave. But at the bottom of wavs, relationship is reflected with the same parameters.
- 4. The blood concentration decreases under the effect of suction process. On contrary blood velocity proportional directly with the effect of suction process.
- 5. The intensity of turbulent flow is decreasing with the increasing of suction parameter b.
- 6. The obtained results of blood velocity and pressure gradient satisfy the Bernoulli concept
- 7. The suction process not only play a dominant parameter in the blood circulation in a horizontal vein, but the majority of diseases need to the suction process as a cupping therapy treatment. Moreover, the suction process in terms of dry and wet cupping therapy solves more than one hundred problems inside the bio tissues of human body.
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- 8. The blood concentration performs lower values under the effect of suction parameter. On contrary, the blood velocity performs higher values in the beginning of suction process.

The importance of suction processes in our practical problems need to more studies for curring many diseases by a nature way as a future prospect of the present work.

CRediT authorship contribution statement:

 M. S. Ali: methodology, software, validation, formal analysis, investigation, resources, data curation, writing original draft preparation, writing review, editing, visualization, supervision and project administration. The author has read and agreed to the published version of the manuscript.

Data availability statement

 The data used to support the findings of this study are available from the corresponding author upon request.

Declaration of competing interest

 The author declare that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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