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Enhancement of the Two-body Nuclear Equation of State in the Framework of BHF Approach

B. M. Mahmoud[1] and A. E. Elmeshneb[1,2,*]

[1] *Physics Department, Faculty of Science, Sohag University, Sohag 82524, Egypt.*
[2] *Department of Physics, College of Science, Yanbu Branch, Taibah University, KSA.*
[*] *Email: Alaaeldeen.elmeshneb@gmail.com*

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Abstract: We have studied some of the bulk and microscopic properties of symmetric nuclear matter within Bruckner-Hartree-Fock approach with angle average approximation and exact Pauli operator using different nucleon-nucleon potentials. The considered potentials in this study are A V_{18} and Nijm I potentials, which give different equations of state. We focused in this work on the influence of Pauli operator treatment on the nuclear matter properties. Our results are good compared with previous studies. However, the empirical saturation point was not achieved. To refine the results, additional approximations were required. Remarkably, introducing three-body forces into the equation of state, particularly using the A V_{18} potential, produced outcomes that closely matched empirical data. Another approach involved adding a correction term to the two-body force, which also resulted in values consistent with empirical observations.
Keywords: Symmetric nuclear matter, Exact Pauli operator, Angle average approximation, Equation of state, BHF approach, TBF.

1. Introduction

In the last decades, nuclear physics has grappled with the intricate task of evaluating the collective and microscopic properties of nuclear matter using a realistic nucleon-nucleon (NN) interaction [1–3]. The equation of state (EOS) governing nuclear matter holds paramount importance in both nuclear physics and astrophysics, fueling a plethora of investigations employing various methodologies [4]. Among these methods, the Bruckner-Hartree-Fock (BHF) approach has emerged as pivotal for probing short-range correlations and nucleon momentum distributions within nuclear matter [5-7]. This method involves solving the two-nucleon equation within the nuclear medium, yielding an energy and density-dependent effective interaction termed the G-matrix. Our analysis concentrates on symmetric nuclear matter and explores three scenarios within the BHF framework. These scenarios encompass employing an angle average approximation of the Pauli operator with a continuous choice of the single-particle potential, utilizing a conventional approach, and finally, employing the exact Pauli operator. BHF approach calculations rely on the selection of the single-particle potential. The conventional choice sets the single-particle energy to zero above the Fermi level [8], whereas the continuous choice presumes that the self-consistent BHF potential expands beyond the Fermi level. In this work we will use the A V_{18} [9] and Nijm I potentials [10]. Even though these potentials forecast nearly identical phase shifts, their mathematical frameworks differ significantly. The Nijm I potential includes terms that depend on momentum, which can be seen as a nonlocal component of the central force, but A V_{18} has a local one. The potential properties explicate the differences in results, as we will show in the next sections. The many-body approach we will use to derive the EOS of symmetric nuclear matter is relatively straightforward, specifically the non-relativistic BHF method. This method employs a conventional and continuous single-particle spectrum and utilizes two contemporary NN potentials. In the present work, we intend to calculate the single particle potential S. P. P. (as a microscopic property) and the EOS of symmetric nuclear matter (as a bulk property), using Computer codes for these Nijm I and A V_{18} Potentials. Discussing the effect of the Pauli operator treatment on the nuclear matter properties is a main aim of this work, moreover the enhancement of the results by introducing two other methods to acquire the saturation properties in nuclear matter.

2. The theoretical framework

The core element of the BHF approach involves the G-matrix, which is defined by the Bethe-Goldstone equation, stated as:

$$G(w) = V + V\frac{Q}{W - H_0 + i\eta} G(w) \quad (1)$$

Here, w represents the initial energy, V denotes the bare two-nucleon potential, H0 stands for the unperturbed energy of intermediate states, η is a small parameter, and Q is the Pauli operator that excludes states with two nucleons beyond the Fermi level. The association is expressed as:

$$Q(k,k^{/}) = (1 - \Theta_F(k))(1 - \Theta_F(k^{/})) \quad (2)$$

In this context, Θ_F (k) = 1 if k is less than kF and zero otherwise and Θ_F represents the probability of occupation for a free Fermi gas where the Fermi momentum k is less than kF.In accordance with the BHF approach, the total energy of nuclear matter is defined by the following equation: $E_A = \sum_k \frac{\hbar^2 k^2}{2m} + \frac{1}{2}\sum_{k^{/}<k_F} < kk^{/} | G(e(k) + e(k^{/})) | kk^{/} >$ (3)

Here, $|kk^{/}\rangle$ denotes to the anti-symmetrization of the G-matrix elements. The single particle energy e(k) is the combination of the kinetic energy T and the single potential energy U(k) and it is expressed as:

$$e(k) = T + U(k) = \frac{\hbar^2 k^2}{2m} + U(k) \quad (4)$$

Here, U(k) is determined by the self-consistent equation as specified in eq. (3):

$$U(k) = \sum_{k^{/}<k_F} < kk^{/}|G(e(k) + e(k^{/}))|kk^{/} > \quad (5)$$

Assuming that the single-particle energy has a quadratic dependence on the nucleon momentum, e(k) can be written in the formula:

$$e(k) = \begin{cases} \frac{\hbar^2 k^2}{2m^*} + \Delta & k \le KF \\ \frac{\hbar^2 k^2}{2m^*} & k > KF \end{cases} \quad (6)$$

where m^* represents the effective mass of the nucleon and Δ is a constant that provides the single-particle energy at k=0.

3. Results and Discussion

In this work, the S. P. P. U (k) for the symmetric nuclear matter using eq. (5) with A V_{18} and Nijm I potentials is calculated with self-consistent method. We plot the reliance of the S. P. P. U (k) on the momentum k in fig. 1 for the considered potentials. From fig. 1, it is observed that U(k) exhibits a straightforward parabolic shape and increases as k rises for two suggested interactions. In addition, it can be observed that the local potential (A V_{18}) is more stiff compared to the non-local potential (Nijm I).

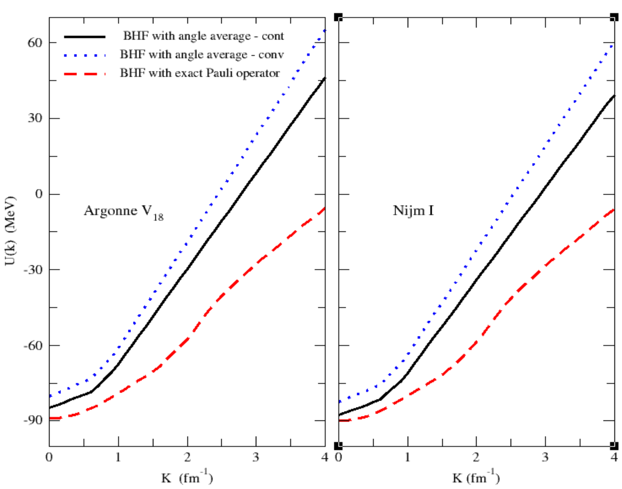

Figure 1: The single particle energy U (k) calculated for symmetric nuclear matter as a function of momentum k.

By computing the depth of the potential (indicating the magnitude of U (k) at k = 0) which equals -80.1611, -84.7308 and -88.9538 MeV for A V_{18} and -82.4072, -87.6437 and -89.9075 MeV for Nijm I, corresponding to the conventional choice, the continuous choice and the exact Pauli operator respectively. Our findings indicate that U(k) exhibits greater repulsion in the angle average approximation compared to the exact Pauli operator. This highlights that the effective interaction between nucleons is more attractive with the exact Pauli operator than with the angle average approximation [11]. We observe that the results are much closer to each other when employing the exact Pauli operator compared to the angle average approximation cases for the two potentials. While the effect of the potential disappears at high momenta in the case of exact Pauli's operator, but still continuous in the angle average approximation cases. In addition, the curve depicting the conventional choice shows a softer and more repulsive behavior compared to the other curves for both potentials. Fig. 2 displays the binding energy per particle EA in MeV against the density ρ for the symmetric nuclear matter using Nijm I potential and comparing with A V_{18} potential from [21]. From the figure, it is clear that the binding energy per nucleon (EA) decreases as the density ρ increases until it reaches a point where it saturates (indicated by solid points). Subsequently, it begins to increase with further increments in ρ.

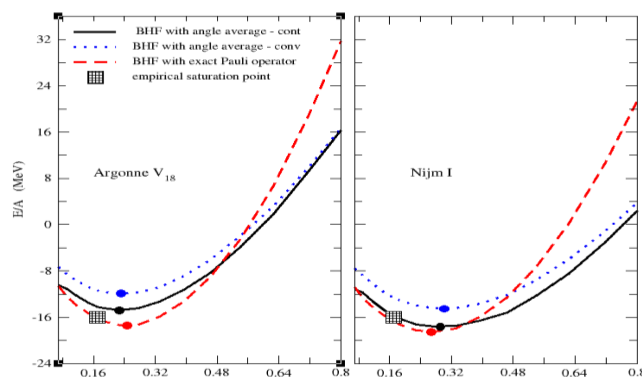

Figure 2: Shows the binding energy per nucleon (EA) for symmetric nuclear matter as a function of density (ρ), with the saturation points marked by solid dots, and the empirical saturation point indicated by a large square. For both Nijm I and A V_{18} potentials.

At low densities, the EA with A V_{18} potential shows stronger repulsion compared to Nijm I. This indicates that the quenching effect, which reduces the influence of non-Born terms in the G-matrix, is less effective for Nijm I than for A V_{18}. Consequently, the Nijm I potential exhibits greater attraction [11]. The saturation points located within a range known as the Coester band [12] shifted with respect to the empirical saturation point ($\rho 0 = 0.17$ fm^{-3}; EA = -16 MeV). Table (1) shows the saturation points in all cases for the two considered potentials. The saturation points extracted in ref. [13] for the same potentials are EA = -17.3 MeV at $\rho 0 = 0.2587$ fm^{-3} for A V_{18} and EA = - 20.7 MeV at $\rho 0 = 0.3452$ fm^{-3} for Nijm I. These differences between the results are attributed to the parabolic shape of the S. P. P. U(k) and the selection of the cut-off momentum set at k =9 fm^{-1} [11].

Table 1 : The values of the saturation binding energy EA and the corresponding density ρ in all cases for the considered potentials.

Model	ρ (fm^{-3})	EA (MeV)
BHF AV_{18} - angle average approx. (cont.)	0.2270	-14.795
BHF AV_{18} - angle average approx. (conv.)	0.2309	-11.902
BHF AV_{18} - Exact Pauli operator	0.2464	-17.398
BHF Nijm I - angle average approx. (cont.)	0.2915	-17.642
BHF Nijm I - angle average approx. (conv.)	0.3009	-14.555
BHF Nijm I - Exact Pauli operator	0.2683	-18.539

As previously stated [14–16], calculations that do not account for relativistic effects and rely solely on two-body interactions have not successfully determined the accurate the point of saturation in symmetric nuclear matter. This recognized shortcoming is typically addressed by incorporating the three-body force (TBF). Although significant advancements have been made in the theoretical understanding of nucleon three-body forces, a comprehensive theory remains undeveloped. The Urbana group has proposed a practical model for nuclear TBF [17]. Specifically, the TBF is expressed as the combination of two distinct components:

$$\boldsymbol{V_{ijk}} = \boldsymbol{V_{ijk}^{2\pi}} + \boldsymbol{V_{ijk}^{R}} \tag{7}$$

The initial term is an attractive component arising from twopion exchange involving the excitement of an intermediate Δ-resonance. Conversely, the second term is a central repulsive phenomenological component. The contribution from the twopion exchange is represented as a cyclic summation across the nucleon indicator i, j, and k, involving products of anticommutator { , } and commutator [,] terms.

$$\boldsymbol{V_{ijk}^{2\pi}} = A \sum cyc \left(\{X_{ij}, X_{jk}\}\{\tau_i.\tau_j\ ,\ \tau_j.\tau_k\}\right.$$
$$\left. + \frac{1}{4}\ [X_{ij}, X_{jk}][\tau_i.\tau_j\ ,\ \tau_j.\tau_k]\ \right), \tag{8}$$

Where

$$X_{ij} = Y(r_{ij})\sigma_i \cdot \sigma_j + T(r_{ij})\ S_{ij} \tag{9}$$

The operator Xij represents the one-pion exchange, where τ and σ represent the isospin and Pauli spin operators, respectively. The tensor operator is defined as $S_{ij} = 3[\ (\sigma_i\ \cdot\ r_{ij})(\sigma_j\ \cdot\ r_{ij}) - \sigma_i\sigma_j\ ]$. The functions T(r) and Y(r) correspond to the tensor and Yukawa functions linked to the one-pion exchange, similar to their role in the two-body potential. The repulsive component is defined as:

$$\boldsymbol{V_{ijk}^{R}} = U \sum cyc\ \ T^2(r_{ij})T^2(r_{ji}) \tag{10}$$

The constants A and U in the preceding equations can be modified to match the observed nuclear properties. We implemented the same Urbana TBF model within the BHF framework, referreed to as BHF+TBF. The resulting equation of state (EOS) derived from using A V_{18} is presented in Table. 2 for symmetric nuclear matter. The EOS saturate at (ρ0 = 0.17 fm−3; EA = -15.6 MeV). It is apparent that the incorporating of the three-body force (TBF) alongside the chosen two-body interaction modifies not just the saturation point but also the entire equation of state (EOS) range for symmetric nuclear matter. Regarding the second potential (Nijm I); The Nijm I potential includes momentum-dependent terms, which result in a non-local potential in configuration space [11]. This non-locality impacts the short-range and medium ingredient of the central force, leading to stronger binding and a softer equation of state (EOS) compared to local potentials like the A V_{18}. Consequently, the reduction of attraction by non-Born terms in the G-matrix is less effective for the Nijm I potential. Due to these characteristics, the evaluation of the Nijm I potential is underestimate the binding energy of three-and four body systems [18,19]. Another approach to achieving saturation properties in nuclear matter involves augmenting the self-energy or effective interaction in BHF calculations with a simple contact interaction (CT). This contact interaction is selected using the notation of the Skyrme interaction [20].

$$\mathrm{CT} = \frac{3}{8}\ t_0\,\rho + \frac{3}{48}\ t_3\,\rho^{1+\delta} \tag{11}$$

In this context, ρ denotes the density, while t_0, t_3, and δ are parameters of the contact interaction. The parameters t_0 and t_3 indicate the zero-range and three-body strength, respectively, and the exponent δ governs the high-density behavior. With δ typically fixed at 0.5, we have adjusted t_0 and t_3 so that the BHF combined with the contact term from eq. (11) aligns with the empirical values of saturation for symmetric nuclear matter (ρ0

= 0.17 fm^{-3}; EA = -16.005 MeV) for the Nijm I potential. The same procedure has done for the BHF using the A V_{18} potential (ρ0 = 0.17 fm^{-3}; EA = -16.0001 MeV) with different values of the parameters t_0 and t_3 as shown in Table 3. For both potentials.

Table 2 : The enhancement of the EOS by adding the correction term for both NN potentials Nijm I and A V_{18} and adding the three-boy force.

ρ (fm^{-3})	BHF Nijm I + CT (MeV)	BHF A V_{18} + CT (MeV)	BHF A V_{18} + TBF (MeV)
0.1484	-15.799	-15.78	-14.6797
0.1699	-16.005	-16.0001	-15.6
0.1853	-15.864	-15.622	-15.015
0.2279	-14.6126	-14.156	-14.198
0.2766	-11.7184	-10.9718	-11.812
0.3318	-6.7749	-5.71008	-7.434
0.3939	0.87731	2.53037	-0.588
0.4633	11.507	13.7925	9.479
0.5403	26.1273	29.190	23.64
0.7192	69.6313	74.515	65.824

Table 3: The parameters t_0, t_3 of the contact interaction (CT), for both Nijm I and A V_{18} potentials.

Parameters	BHF Nijm I	BHF A V_{18}
t_0 [MeV fm^3]	-184.2	-167.76
t_3 [MeV $fm^{4.5}$]	2850	2499

4. Conclusion

Finally, it is noted that the precise treatment of the Pauli operatorresults in enhanced correlation effects in the medium compared to the angle average approximation with his two choices. Nevertheless, the impact of this approach is relatively minor near the empirical saturation point at low densities, even when varying the potential used, consistent with earlier research findings [8]. However, the accurate determination of the saturation point remains elusive, indicating a necessity for adjustments in the model to address this issue. This suggests a need for model adjustments to resolve the issue. Modifications were implemented using two approaches: first, by incorporating the three-body force (TBF) into the local A V_{18} potential. The integration of TBF with the existing two-body forces significantly altered the overall trend. Without requiring precise adjustments, the resulting equation of state achieved a saturation method involves introducing a contact interaction (CT) into our Brueckner-Hartree-Fock (BHF) framework to improve the two -body equation of state. This adjustment yields empirical values and enhances the accuracy of the equation of state across a wide range of densities. Finally, when the saturation point is consistently reproduced with precision, it signifies a robust. determination of the complete equation of state (EOS). The nuclear EOS can be considered uniquely defined with a high degree of accuracy, particularly when employing microscopic many-body theory and realistic, precise two-body interactions.

CRediT authorship contribution statement:

"Conceptualization, A. E. Elmeshneb and B. M. Mahmoud; methodology, A. E. Elmeshneb; software, A. E. Elmeshneb; validation, A. E. Elmeshneb and B. M. Mahmoud; formal analysis, A. E. Elmeshneb and B. M. Mahmoud; investigation, A. E. Elmeshneb and B. M. Mahmoud; resources, A. E. Elmeshneb; data curation, A. E. Elmeshneb; writing—original draft preparation, B. M. Mahmoud; writing—review and editing, A. E. Elmeshneb; visualization, A. E. Elmeshneb and Basma M. Elyan; supervision, A. E. Elmeshneb; project administration, A. E. Elmeshneb and B. M. Mahmoud; funding acquisition, A. E. Elmeshneb All authors have read and agreed to the published version of the manuscript.

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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